

Induction of the higher-derivative Chern-Simons extension in QED₃

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We perform the perturbative generation of the higher-derivative Chern-Simons contribution to the effective action in the three-dimensional QED at zero and finite temperature. In the latter case, we show that as the temperature goes to infinity this contribution vanishes. However, as expected, as the temperature goes to zero only the covariant part survives. The non-covariant part contributes only in intermediate temperatures where presents a maximum.

I. INTRODUCTION

The higher-derivative contributions to the effective action actually call the attention within supersymmetry, gravity, Lorentz-breaking theories and many other contexts. Originally, their study has been motivated by consideration of the gravity models where, because of the non-renormalizability of the Einstein gravity, the hopes for development of the theory consistent at the quantum level have been initially related namely with introduction of the higher derivatives [1]. Despite the difficulties displayed by these theories at the quantum level such as arising of ghosts, see e.g. [2], these theories became an important tool for treating the low-energy effective behaviour, especially in the gravity (for many examples, see the book [3]), and also in theory of strings and higher spins [4]. Moreover, some efficient manners to deal with these theories have been proposed by Trodden and Fontanini [5] who used a completely Euclidean description for the higher-derivative models, and by Smilga [6] who proposed a way to control the impact of ghosts. However, up to now, most studies for the higher-derivative theories (including supersymmetric and Lorentz-breaking contexts) have been carried out only in four-dimensional space-time whereas their consid-

eration in three dimensions seems to be very interesting – first, due to the great attention to the three-dimensional field theories inspired by studies of the graphene, second, due to the well-known one-loop finiteness (and, of course, general improvement of the renormalization behavior) and overall simplicity of the three-dimensional field theories. Furthermore alternative higher-derivative gravity models based on different scalings of space and time coordinates have been addressed within the Horava-Lifshitz approach [7]. In this case the higher derivatives are present only in the spatial sector thus eliminating the ghost problem, and the theory turns out to be consistently renormalizable by power counting. At the same time, the critical exponent responsible characterizing the degree of higher spatial derivatives can flow in such a way that the theory effectively describes lower-dimensional physics in high energy limit. So, example of Horava-Lifshitz theories is two-fold. It presents renormalizability of the quantum theory, and in the same time it effectively flows to a lower dimensional theory. Thus, it seems to be important to look for other theories with similar behavior, i.e. to search for higher-derivative theories at lower dimensions. This is the problem we shall address in this paper.

We start with the scheme allowing for inducing a higher derivative Chern-Simons extension via the QED₃. The only one such Lorentz-invariant extension is given by [8, 9]

$$\mathcal{S}_{ECS} = (2m)^{-1} \kappa \int d^3x \varepsilon^{\alpha\beta\rho} A_\alpha \partial_\beta \square A_\rho, \quad (1)$$

where m is the mass of theory and $\kappa \propto 1/|m|$ up to a numerical factor that we shall determine through radiative induction of this term. Notice that by using integration by parts in the above expression we find

$$\mathcal{S}_{ECS} = -(2m)^{-1} \kappa \int d^3x \varepsilon^{\alpha\beta\rho} \tilde{A}_\alpha \partial_\beta \tilde{A}_\rho \quad \text{with} \quad \tilde{A}_\alpha = \frac{1}{2} \varepsilon_{\alpha\mu\nu} F^{\mu\nu}, \quad (2)$$

which depends locally on the field strength and *not* on the potential.

The main purpose of this paper consists in inducing the term (2) as a radiative correction through the use of the QED₃ action by integrating one loop fermionic field at zero and at finite temperature. The paper is organized as follows: in the section II, we use the derivative expansion scheme and apply it to induce the higher Chern-Simons actions at zero temperature. Then we proceed in the sections III, IV with studying of their thermal dependence using the Matsubara formalism for fermions. Finally in section V we present our conclusions.

II. THE ONE-LOOP EFFECTIVE ACTION

In order to perform this calculation we consider the following fermionic part of the QED₃ Lagrangian

$$\mathcal{L}_\psi = \bar{\psi}(i\cancel{\partial} - m)\psi - q\bar{\psi}\cancel{A}\psi. \quad (3)$$

We note that here $\cancel{A} = \gamma^\mu A_\mu$, where A_μ is only an usual gauge field. In principle, our results can be straightforwardly generalized to yield more sophisticated higher-derivative terms, if we replace the A_μ by any vector linear in A_μ , such as, for example, $\square A_\mu$, $\partial^\lambda F_{\lambda\mu}$, etc., just in this manner some of contributions in [13] has been calculated. We can integrate out the fermion field ψ in the functional integral to obtain the one loop effective action for the A_μ field that reads

$$S_{eff}[A_\mu(x)] = -i \text{Tr} \ln[\cancel{p} - m - q\cancel{A}(x)], \quad (4)$$

where the symbol Tr stands for the trace over Dirac matrices, trace over the internal space as well as for the integrations in momentum and coordinate spaces. The constant factor has been absorbed into normalization of the path integral such that

$$S_{eff}[A_\mu(x)] = S_{eff} + S_{eff}^n[A_\mu(x)], \quad (5)$$

where

$$S_{eff}^n[A_\mu(x)] = i \text{Tr} \sum_{n=1} \frac{1}{n} \left[-iq S_f(p) \cancel{A}(x) \right]^n \quad (6)$$

and

$$S_f(p) = \frac{i}{\cancel{p} - m} \quad (7)$$

is the usual fermion propagator associated with the theory.

Let us now consider the terms for $n = 2$ of the power expanded logarithm in Eq. (7) to write

$$S_{eff}^{n=2}[A_\mu(x)] = -\frac{iq^2}{2} \text{Tr} [S_f(p) \cancel{A}(x) S_f(p) \cancel{A}(x)]. \quad (8)$$

Now applying the main property of derivative expansion method, we observe that any function of momentum can be converted into a coordinate dependent quantity as [10]

$$A(x) S_f(p) = (S_f(p - i\partial) A(x)). \quad (9)$$

The parenthesis on the right hand side merely emphasizes that the derivatives act only on $A(x)$. In this case, the expression (8) becomes

$$S_{eff}^{n=2}[A_\mu(x)] = -\frac{iq^2}{2}\text{Tr}[S_f(p)\gamma^\alpha(S_f(p-i\partial)A_\alpha)A]. \quad (10)$$

By considering the following expansion

$$S_f(p-i\partial) = S_f(p) + S_f(p)\not{\partial}S_f(p) + S_f(p)\not{\partial}S_f(p)\not{\partial}S_f(p) + S_f(p)\not{\partial}S_f(p)\not{\partial}S_f(p)\not{\partial}S_f(p) + \dots \quad (11)$$

the effective action (10) can be rewritten in the form

$$S_{eff}^{n=2}[A_\mu(x)] = -\frac{iq^2}{2} \int d^3x \left[I^{\alpha\mu\nu\lambda\beta} (\partial_\mu \partial_\nu \partial_\lambda A_\alpha) A_\beta \right], \quad (12)$$

where the quantity $I^{\alpha\mu\nu\lambda\beta}$ is given by

$$I^{\alpha\mu\nu\lambda\beta} = \int \frac{d^3p}{(2\pi)^3} \text{tr}[S_f(p)\gamma^\alpha S_f(p)\gamma^\mu S_f(p)\gamma^\nu S_f(p)\gamma^\lambda S_f(p)\gamma^\beta] \quad (13)$$

and the symbol tr denotes the trace of the product of the gamma matrices.

Now, we substitute here the expression of the fermion propagator (7) together with the following trace identity: $\text{tr}[\gamma^\mu \gamma^\nu \gamma^\rho] = 2i\varepsilon^{\mu\nu\rho}$. In this case, we can rewrite the expression (13) as

$$I^{\alpha\mu\nu\lambda\beta} = -2m\varepsilon^{\alpha\lambda\beta} \int \frac{d^3p}{(2\pi)^3} \left[\frac{((p^2 - m^2)\eta^{\mu\nu} - 4p^\mu p^\nu)}{(p^2 - m^2)^4} \right]. \quad (14)$$

The results for relevant momentum integrals, within the dimensional regularization approach, are:

$$\int \frac{d^3p}{(2\pi)^3} \frac{1}{(p^2 - m^2)^n} = \frac{(-1)^n i}{(4\pi)^{3/2}} \frac{\Gamma(n-3/2)}{\Gamma(n)} \frac{1}{(m^2)^{n-3/2}} \xrightarrow{n=3} \frac{-i}{8\pi} \frac{1}{4m^2|m|}, \quad (15)$$

$$\int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{(p^2 - m^2)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{3/2}} \frac{\eta^{\mu\nu}}{2} \frac{\Gamma(n-5/2)}{\Gamma(n)} \frac{1}{(m^2)^{n-5/2}} \xrightarrow{n=4} \frac{-i}{8\pi} \frac{\eta^{\mu\nu}}{24m^2|m|}. \quad (16)$$

As a result, we obtain

$$I^{\alpha\mu\nu\lambda\beta} = \frac{i}{8\pi} \frac{1}{6m|m|} \varepsilon^{\alpha\lambda\beta} \eta^{\mu\nu}. \quad (17)$$

Therefore, the expression (12) becomes

$$S_{eff}^{n=2}[\tilde{A}_\mu] = -(2m)^{-1}\kappa \int d^3x \varepsilon^{\alpha\mu\beta} \tilde{A}_\alpha \partial_\mu \tilde{A}_\beta, \quad \kappa = \frac{1}{192\pi} \frac{q^2}{|m|}. \quad (18)$$

This is just the term whose form has been proposed in the Introduction. It is worth to mention that if we abandon the requirement of Lorentz invariance, the structure of the higher-derivative contributions to the effective action can be richer, for example, in the Horava-Lifshitz-like extension of (2+1)-dimensional QED, the higher-derivative contributions to the effective action have been found in [12], in principle, one can also try to find the three-dimensional analogues of the results obtained in [13], where it is natural to expect the CPT-even Lorentz-breaking terms; however, the action we use essentially differs from that one considered in [12, 13].

III. HIGHER DERIVATIVE CHERN-SIMONS AT FINITE TEMPERATURE

We shall now make use of imaginary time formalism to induce the higher derivative Chern-Simons term at finite temperature. In this case, we change the Minkowski space to Euclidean one by performing the Wick rotation: $x_0 \rightarrow ix_0$, $p_0 \rightarrow ip_0$ such that $d^3x \rightarrow id^3x$, $d^3p \rightarrow id^3p$. Thus, we substitute the self-energy tensor (14) in the effective action (12) such that the Euclidean resulted action is given by

$$S_{eff}^{Euc}[A_\mu(x)] = imq^2 \int d^3x \varepsilon^{\alpha\lambda\beta} (\partial_\mu \partial_\nu \partial_\lambda A_\alpha) A_\beta \int \frac{d^3p}{(2\pi)^3} \left[\frac{((p^2 + m^2)\delta^{\mu\nu} - 4p^\mu p^\nu)}{(p^2 + m^2)^4} \right]. \quad (19)$$

To develop calculations with finite temperature, let us now assume that the system is in the state of the thermal equilibrium with a temperature $T = 1/\beta$. In this case we can use the Matsubara formalism for fermions. This consists of taking $p_0 \rightarrow \omega_n = (n + 1/2)2\pi/\beta$ and replacing the integration over zeroth component of the momentum by a discrete sum over n : $(1/2\pi) \int dp_0 \rightarrow 1/\beta \sum_n$ [11]. Additionally, we implement translation only on the space coordinates of the loop momentum p_μ and we decompose it as follows: $p_\mu \rightarrow \vec{p}_\mu + p_0 \delta_{0\mu}$ [15]. This leads us to the identity

$$\vec{p}_\mu \vec{p}_\nu \rightarrow \frac{\vec{p}^2}{2} (\delta_{\mu\nu} - \delta_{0\mu} \delta_{0\nu}). \quad (20)$$

Here, we use the covariance under spatial rotations. Now, we apply all the above information in the effective action (19), therefore the temperature-dependent effective action turns out

to reproduce the following structure:

$$S_{eff}^{Euc}[A_\mu(x)] = \frac{imq^2}{\beta} \int d^3x \left[\varepsilon^{\alpha\lambda\beta} (\partial^2 \partial_\lambda A_\alpha) A_\beta \sum_{n=-\infty}^{+\infty} \int \frac{d^2\vec{p}}{(2\pi)^2} \left[\frac{1}{(\vec{p}^2 + M_n^2)^3} - \frac{2\vec{p}^2}{(\vec{p}^2 + M_n^2)^4} \right] - 4\varepsilon^{\alpha\lambda\beta} (\partial_0^2 \partial_\lambda A_\alpha) A_\beta \sum_{n=-\infty}^{+\infty} \int \frac{d^2\vec{p}}{(2\pi)^2} \left[\frac{\frac{\vec{p}^2}{2} - M_n^2 + m^2}{(\vec{p}^2 + M_n^2)^4} \right] \right]. \quad (21)$$

where

$$M_n^2 = \left(n + \frac{1}{2}\right)^2 \frac{4\pi^2}{\beta^2} + m^2. \quad (22)$$

After performing the momentum integration, we arrive at

$$S_{eff}^{Euc}[A_\mu(x)] = \frac{iq^2}{16\pi^2} \frac{a^3}{m|m|} \int d^3x \left[\frac{1}{3} \varepsilon^{\alpha\lambda\beta} (\partial^2 \partial_\lambda A_\alpha) A_\beta \sum_{n=-\infty}^{+\infty} \frac{1}{[(n + \frac{1}{2})^2 + a^2]^2} + 2\varepsilon^{\alpha\lambda\beta} (\partial_0^2 \partial_\lambda A_\alpha) A_\beta \times \left[\sum_{n=-\infty}^{+\infty} \frac{1}{[(n + \frac{1}{2})^2 + a^2]^2} - \frac{4}{3} \sum_{n=-\infty}^{+\infty} \frac{a^2}{[(n + \frac{1}{2})^2 + a^2]^3} \right] \right], \quad (23)$$

where $a = \frac{m\beta}{2\pi}$. Now, we use the well-known results for sums over the Matsubara frequencies (see f.e. [14]):

$$\begin{aligned} \sum_{n=-\infty}^{+\infty} \frac{1}{[(n + \frac{1}{2})^2 + a^2]^2} &= -\frac{\pi}{a^3} \left[\pi a \operatorname{sech}^2(\pi a) - \tanh(\pi a) \right], \\ \sum_{n=-\infty}^{+\infty} \frac{a^2}{[(n + \frac{1}{2})^2 + a^2]^3} &= \frac{\pi}{8a^3} \left[3\tanh(\pi a) - \pi a \operatorname{sech}^2(\pi a) (3 + 2\pi a \tanh(\pi a)) \right] \end{aligned} \quad (24)$$

into the expression (23) to find

$$S_{eff}^{Euc}[A_\mu(x)] = \frac{iq^2}{8\pi} \frac{1}{6m|m|} \int d^3x \left[\varepsilon^{\alpha\lambda\beta} (\partial^2 \partial_\lambda A_\alpha) A_\beta f(a) + \varepsilon^{\alpha\lambda\beta} (\partial_0^2 \partial_\lambda A_\alpha) A_\beta g(a) \right], \quad (25)$$

where the quantities $f(a)$ and $g(a)$ are ‘thermal functions’ given respectively by

$$\begin{aligned} f(a) &= -\pi a \operatorname{sech}^2(\pi a) + \tanh(\pi a), \\ g(a) &= 2(\pi a)^2 \tanh(\pi a) \operatorname{sech}^2(\pi a). \end{aligned} \quad (26)$$

They comprise the covariant and non-covariant contributions, respectively. The asymptotic analysis of the above thermal functions yields the following results

$$\begin{aligned} f(a \rightarrow 0) &\rightarrow 0 \quad (T \rightarrow \infty), \quad f(a \rightarrow \infty) \rightarrow 1 \quad (T \rightarrow 0); \\ g(a \rightarrow 0) &\rightarrow 0 \quad (T \rightarrow \infty), \quad g(a \rightarrow \infty) \rightarrow 0 \quad (T \rightarrow 0). \end{aligned} \quad (27)$$

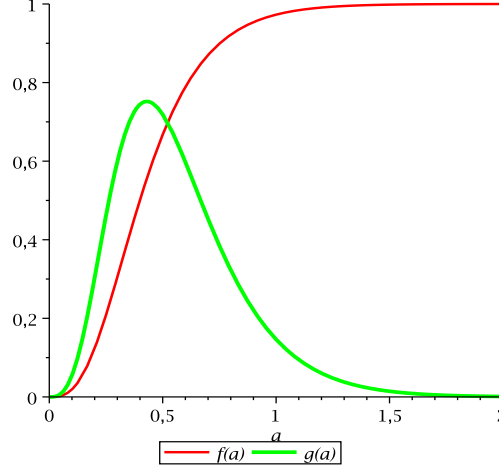


FIG. 1: The behavior of $f(a)$ and $g(a)$, corresponding to covariant and non-covariant contributions, respectively. Since $a = \frac{m\beta}{2\pi}$, as the temperature is very high these functions vanish. As expected, at very low temperature only the covariant part survives. The non-covariant contribution presents a maximum in intermediate temperatures.

The full behavior is depicted in Fig. 1. Recalling that $a = \frac{m\beta}{2\pi}$, we see that as the temperature is very high these functions vanish. However, as the temperature goes to zero only the covariant part of (25) survives and recovers the result obtained for zero temperature (18), as expected. It is interesting to note that the non-covariant part contributes only in intermediate temperatures presenting a maximum for some intermediate temperature.

IV. THERMAL EFFECTIVE ACTION

The resulted (24) displays the possibility to construct a thermal higher-derivative theory. The associated quantum correction can now be written as the gauge fixing term, so, in the Minkowski space the effective Lagrangian is

$$\mathcal{L}_{thm} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\tilde{\kappa}}{2m}\varepsilon^{\mu\beta\nu}(\partial_\beta A_\mu)(\partial^2 f(a) + \partial_0^2 g(a))A_\nu - \frac{1}{2\xi}(\partial \cdot A)^2, \quad (28)$$

where $\tilde{\kappa} = q^2/24\pi|m|$. Here, we want to use the Lagrangian (28) to investigate the effects of temperature on the gauge excitations. A way of study this matter is based on the associated propagator. Thus, we can immediately write the form of our propagator in usual form corresponding to the Maxwell-Chern-Simons action (in Feynman gauge $\xi = 1$),

$$\Delta^{\mu\nu} = -[p^2 - m^2]^{-1}(g^{\mu\nu} - i(m/p^2)\varepsilon^{\mu\alpha\nu}p_\alpha), \quad (29)$$

with the following redefinition $m \rightarrow \tilde{\kappa}/m(p^2 f(a) + p_0^2 g(a))$ is carried out in the momentum space. The resultant thermal propagator is given by

$$\Delta_{th}^{\mu\nu} = \left[p^2 - (m^2)^{-1} (p^2 f(a) + p_0^2 g(a))^2 \tilde{\kappa}^2 \right]^{-1} \left(g^{\mu\nu} - i(m p^2)^{-1} \times \right. \\ \left. \times (p^2 f(a) + p_0^2 g(a)) \tilde{\kappa} \varepsilon^{\mu\alpha\nu} p_\alpha \right). \quad (30)$$

Notice that the denominator associated to propagator (30) describes two groups of gauge excitations,

$$\frac{1}{\left[p^2 - (m^2)^{-1} (p^2 f(a) + p_0^2 g(a))^2 \tilde{\kappa}^2 \right]} = -\frac{1}{p^2} + \frac{1}{p^2 \left(1 - \frac{m^2 p^2}{(p^2 f(a) + p_0^2 g(a))^2 \tilde{\kappa}^2} \right)} \quad (31)$$

which one group is massless and the other are massive and thermal given by

$$E_\pm = \pm \frac{\sqrt{\lambda_1 \pm \sqrt{\lambda_1^2 - \lambda_2 (h(a))^2}}}{h(a)} \quad (32)$$

where $h(a) = f(a) + g(a)$ and the quantities: λ_1 and λ_2 are given by

$$\lambda_1 = f(a)h(a)|\vec{p}|^2 + \frac{\tilde{m}^2}{2} \quad (33)$$

$$\lambda_2 = |\vec{p}|^2 (|\vec{p}|^2 + \tilde{m}^2), \quad \text{with} \quad \tilde{m} = m/\kappa. \quad (34)$$

Notice that in the limit of temperature goes to zero ($T \rightarrow 0$) the massive-gauge excitations are restored. However, in the massless case, the gauge excitations remain dependent on thermal functions.

V. CONCLUSION

We have showed how the higher-derivative terms can be generated at the one-loop order in the three-dimensional QED at zero and finite temperature. In the latter case, we have considered a covariant and non-covariant part. They display different behaviour. Whereas the covariant part is recovered as the temperature goes to zero, the non-covariant part has a maximum only in intermediate temperatures. On the other hand, both parts go to zero as the temperature goes to infinity. The fact that the covariance is strongly broken by the non-covariant part in some intermediate scale of high temperature (see Fig. 1) may set a scale of covariance breaking in theories where spatial and temporal momentum components scales differently, such as in Hořava-Lifshitz gravity [7].

The natural continuation of the study we performed here could consist in generalization of our results for non-Abelian and Lorentz-breaking cases (it worth to mention that, up to now, the number of known Lorentz-breaking results in three-dimensional space-time is very small). Also, this methodology can be applied as well to generating of higher-derivative corrections in linearized gravity and to obtaining more sophisticated quantum corrections in the Horava-Lifshitz-like case generalizing the results of [12].

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